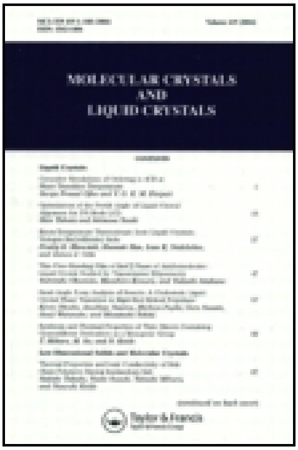


Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

# Color Liquid Crystal Display - Problems and Optimization Procedure

Jerzy Zieliński [a] & Marek Olifierczuk [a]
[a] Institute of Applied Physics, Military University of Technology, Warsaw, Poland

Version of record first published: 22 Sep 2006

To cite this article: Jerzy Zieliński & Marek Olifierczuk (2006): Color Liquid Crystal Display - Problems and Optimization Procedure, Molecular Crystals and Liquid Crystals, 454:1, 387/[789]-405/[807]

To link to this article: http://dx.doi.org/10.1080/15421400600656277

*Mol. Cryst. Liq. Cryst.*, Vol. 454, pp. 387/[789]–405/[807], 2006

ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400600656277





# Color Liquid Crystal Display – Problems and Optimization Procedure

**Jerzy Zieliński**
**Marek Olifierczuk**
Institute of Applied Physics, Military University of Technology, Warsaw, Poland

*In this work the mathematical model of a light propagation through the system of layers with dichroic properties are presented. This model makes it possible to obtain the spectral characteristics of transmission and reflection for any system of isotropic, anisotropic and dichroic layers. Additionally, the calculations takes into account real conditions of a display working. Basing of this model the computer program is done. This program makes it possible to conduct the complete optimization procedure for very complicated system such as for example color liquid crystals display.*

**Keywords:** mathematical model of light propagation; numerical calculations; optimization process of LCD

## 1. INTRODUCTION

The aim of our work is presenting new and complete mathematical model of a light propagation through the system of layers, which can be layers with isotropic, anisotropic or dichroic properties. This model takes into account the real properties of the display elements such as dispersion phenomena of layers refractive index and complex form of it. Additionally, the interference phenomena occurred into the display, spectral characteristic of a light source and human eye sensitivity is taken into account. Except of it, there makes it possible to do the calculations for any observation angle. This model can be used in the analysis or optimization process of very complicated layers system such as a color liquid crystal displays. It should be underlined, that

Address correspondence to Marek Olifierczuk, Institute of Applied Physics, Military University of Technology, 2 Kaliskiego St., 00-908 Warsaw, Poland. E-mial: molifierczuk@wat.edu.pl

presented theory is worked out without any simplifications. Such a theory can be a base for computer program to calculate the real optical parameters of a display and can be use to provide the complete and effective optimization procedure.

The calculation done in this work showed that the construction process of color liquid crystal display is more complicated than black and white display, because there are more problems with obtaining proper visualization quality. In this case there is very important to obtain not only high contrast ratio and brightness of the display, but also the color has to be visualized in proper way. In the other words, the optimization process should be done for different wavelengths and the final results should take into account the color detection properties of a human eye. Therefore, the choice of an optical matching point of a liquid crystal layer and optical properties of the used other display elements should take into account these assumptions. For this reason, it is necessary to have a calculation's tool, which can give us the close information about display optical parameters for any set of the display elements.



## 2. NUMERICAL BASE OF THE DISPLAY'S OPTICAL PARAMETERS CALCULATIONS

The base of our calculation method are Maxwell equations, which for nonmagnetic and nonabsorbing media ($\sigma = 0$ and $\rho = 0$) can be written as:

$$\begin{aligned} &1)\ \mathrm{rot}\,\overrightarrow{H} - \frac{\delta\overrightarrow{D}}{\delta t} = 0 \\ &2)\ \mathrm{rot}\,\overrightarrow{E} + \frac{\delta\overrightarrow{B}}{\delta t} = 0 \quad \text{and} \quad \begin{aligned} \overrightarrow{D} &= \varepsilon_o \overset{\vee}{\varepsilon} \overrightarrow{E} \\ \overrightarrow{B} &= \mu_o \mu \overrightarrow{H} \end{aligned} \\ &3)\ \mathrm{div}\,\overrightarrow{D} = 0 \\ &4)\ \mathrm{div}\,\overrightarrow{B} = 0 \end{aligned} \tag{1}$$

where $\overset{\vee}{\varepsilon}$ denotes dielectric tensor in principle coordinate system $O^G(xG, yG, zG)$:

$$\overset{\vee}{\varepsilon} = \begin{bmatrix} \varepsilon_{or} & 0 & 0 \\ 0 & \varepsilon_{or} & 0 \\ 0 & 0 & \varepsilon_{ex} \end{bmatrix} \tag{2}$$

$\varepsilon_{or}$ and $\varepsilon_{ex}$ denote here dielectric permittivity for perpendicular and parallel directions to the optical axis of a layer [1,2].

Solving Maxwell equations (1) and taking into account the wavevector of a light in coordinate system $O^G$ in the form $\vec{k} = [k_{xG}, k_{yG}, k_{zG}]$, one can obtain:

$$\begin{bmatrix} \frac{\omega^2}{c^2}n_o^2 - k_{yG}^2 - k_{zG}^2 & k_{xG}k_{yG} & k_{xG}k_{zG} \\ k_{xG}k_{yG} & \frac{\omega^2}{c^2}n_o^2 - k_{xG}^2 - k_{zG}^2 & k_{yG}k_{zG} \\ k_{xG}k_{zG} & k_{yG}k_{zG} & \frac{\omega^2}{c^2}n_e^2 - k_{xG}^2 - k_{yG}^2 \end{bmatrix} \begin{bmatrix} E_{xG} \\ E_{yG} \\ E_{zG} \end{bmatrix} = 0 \tag{3}$$

where $n_o$ and $n_e$ denote the refractive ordinary and extraordinary index, respectively [2].

The Eq. (3) has nontrivial solution only where the determinant of a left matrix vanish, so the solution of it makes two equations:

$$\left(\frac{k_{xG}^2 + k_{yG}^2}{n_e^2} + \frac{k_{zG}^2}{n_o^2} - \frac{\omega^2}{c^2}\right) = 0 \quad \text{and} \quad \left(\frac{k^2}{n_o^2} - \frac{\omega^2}{c^2}\right) = 0 \tag{4}$$

where $k^2 = k_{xG}^2 + k_{yG}^2 + k_{zG}^2$.

As one can see from Eq. (4) in given direction $\vec{s}$ we have simultaneously two waves:

- the first one with a wavevector fulfilling relation $k_o^2 = (\omega^2/c^2)n_o^2$, which module do not depend on a direction of propagation. Therefore, this wave is called ordinary wave, wavevector $\vec{k_o}$ – wavevector of an ordinary wave and refractive index $n_o$ – ordinary refractive index.
- the second wave with a wavevector $\vec{k_e}$, which the coordinates fulfill the left part of Eq. (4) and which module depends on a direction of propagation. This wave is called extraordinary wave, wavevector $\vec{k_e}$ – wavevector of an extraordinary wave and coefficient $n_e$ – extraordinary refractive index.

After determination of wavevectors the polarization of its can be fixed. Using the Eq. (3) for ordinary and extraordinary waves one can show, that unit vectors of electric field for these wave have the following forms:

$$\overrightarrow{o_G} = \left[\frac{-k_{yG}}{\sqrt{k_{xG}^2 + k_{yG}^2}}, \frac{k_{xG}}{\sqrt{k_{xG}^2 + k_{yG}^2}}, 0\right] \tag{5}$$

$$\overrightarrow{e_G} = \left[\frac{k_{xG}}{M\left(\frac{\omega^2 n_o^2}{c^2} - k^2\right)}, \frac{k_{yG}}{M\left(\frac{\omega^2 n_o^2}{c^2} - k^2\right)}, \frac{k_{zG}}{M\left(\frac{\omega^2 n_e^2}{c^2} - k^2\right)}\right] \tag{6}$$

where

$$M = \sqrt{\frac{k_{xG}^2 + k_{yG}^2}{\left(\frac{\omega^2 n_o^2}{c^2} - k^2\right)^2} + \frac{k_{zG}^2}{\left(\frac{\omega^2 n_e^2}{c^2} - k^2\right)^2}}.$$

As one can see the analysis presented above give us the information about the forms of wavevectors generated in the medium for given direction of a light propagation and the forms of unit vectors of $\vec{E}$ connected with these wavevectors. It should be underlined, that these information are obtained for principle coordinate system of a layer.

It is easy to show, that an analysis done for an anisotropic layer in the form presented above can be revised in very similar way for a dichroic layer. In this case the electric conductivity $\sigma \neq 0$ and can be assumed as tensor in the following form:

$$\overset{\vee}{\sigma} = \begin{bmatrix} \sigma_{or} & 0 & 0 \\ 0 & \sigma_{or} & 0 \\ 0 & 0 & \sigma_{ex} \end{bmatrix} \tag{7}$$

where $\sigma_{or}$ and $\sigma_{ex}$ denote the values of conductivity for perpendicular and parallel direction to the optical axis of layer, respectively.

Solving Maxwell equations for this case the following relations for ordinary and extraordinary wavevectors and unit vectors of $\vec{E}$ can be obtained:

- for ordinary wave the wavevector fulfills the relation $(\hat{k}_o^2/(n_o - i\chi_o)^2) - (\omega^2/c^2) = 0$ and the unit vector of an electric intensity $\vec{E}$ has a form:

$$\overrightarrow{\hat{o}_G} = \left[\frac{-\hat{k}_{oyG}}{\sqrt{\left|\hat{k}_{oxG}\right|^2 + \left|\hat{k}_{oyG}\right|^2}}, \frac{\hat{k}_{oxG}}{\sqrt{\left|\hat{k}_{oxG}\right|^2 + \left|\hat{k}_{oyG}\right|^2}}, 0\right] \tag{8}$$

- for extraordinary wave the wavevector fulfills the relation $((\hat{k}_{exG}^2 + \hat{k}_{eyG}^2)/(n_e - i\chi_e)^2) + (\hat{k}_{ezG}^2/(n_o - i\chi_o)^2) - (\omega^2/c^2) = 0$ and the unit vector of $\vec{E}$ is equal:

$$\overrightarrow{\hat{e}_G} = \left[\frac{\hat{k}_{exG}}{M\left(\frac{\omega^2(n_o - i\chi_o)^2}{c^2} - \hat{k}_e^2\right)}, \frac{\hat{k}_{eyG}}{M\left(\frac{(\omega^2 n_o - i\chi_o)^2}{c^2} - \hat{k}_e^2\right)}, \frac{\hat{k}_{ezG}}{M\left(\frac{\omega^2(n_e - i\chi_e)^2}{c^2} - \hat{k}_e^2\right)}\right] \tag{9}$$

where

$$M = \sqrt{\frac{\left|k_{exG}\right|^2 + \left|k_{eyG}\right|^2}{\left|\frac{\omega^2(n_o - i\chi_o)^2}{c^2} - \overset{\wedge}{k_e}^2\right|^2} + \frac{\left|k_{ezG}\right|^2}{\left|\frac{\omega^2(n_e - i\chi_e)^2}{c^2} - \overset{\wedge}{k_e}^2\right|^2}}.$$

As one can see in the equations presented above the refractive indices, wavevectors and unit vectors of $\overrightarrow{E}$ have the complex representations. The refractive indices are equal:

$$\begin{aligned} \overset{\wedge}{n}_o &= n_o - i\chi_o \\ \overset{\wedge}{n}_e &= n_e - i\chi_e \end{aligned} \tag{10}$$

and the wavevectors have a form $k = (\omega/c)(n - i\chi)$. It can be shown that the coefficient $\chi$ is responsible for absorption [3,4]. The absorption coefficient can be defined as $\alpha = 2\,(\omega/c)\chi$.

One can notice, that the units vectors of $\overrightarrow{E}$ for specific directions (perpendicular and parallel to the optical axis of a layer) can not be determined using the equations obtained above, and yet its can be determined directly from Maxwell equations described for these directions.

The relations presented above for the wavevectors and unit vectors of $\overrightarrow{E}$ are described in principle coordinate system of a given layer, but its make it possible to describe similar relations for the coordinate system common for all layers of a display. Every layer has an own principle coordinate system $O^G(xG, yG, zG)$, which is connected with it's optical axis. This system has unit vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ ($\vec{c}$ – optical axis direction). We can introduce two other coordinate systems: the first one connected with laboratory $O^L(xL, yL, zL)$, with unit vectors $\vec{x}'$, $\vec{y}'$ and $\vec{z}'$ (it is constant system and $\vec{z}'$ is perpendicular to a layers surface) and the second one called *surface system* (which is connected with incident wave) denoted as $O^S(xS, yS, zS)$ with unit vectors $\vec{x}$, $\vec{y}$ and $\vec{z}$. This system is dependent on a direction of a wavevector in external medium. The unit vector $\vec{z}$ is according to the unit vector $\vec{z}'$. The mutual relations between these systems are presented in Figure 1.

The following assumption were established in the construction of these systems:

- unit vector $\vec{c}$ is lied according to the optical axis of a layer and is turned to positive half-space of system $O^L$;
- unit vector $\vec{a}$ lie in surface xLxG, and unit vector $\vec{b} = \vec{c} \times \vec{a}$;
- angle $\alpha$ is positive for rotation of a system $O^P$ in the direction shown in Figure 1;
- wavevector in external medium is lied in surface xPzP of surface system.

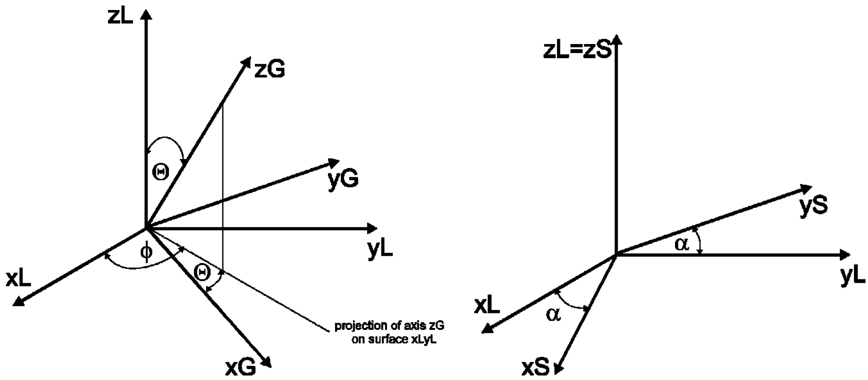


**FIGURE 1** The principle, laboratory and surface coordinate systems.

For these systems the following transposition matrixes can be described:

$$
\begin{aligned}
M(O^L \to O^G) &= \begin{bmatrix} \cos\phi\cos\Theta & \sin\phi\cos\Theta & -\sin\Theta \\ -\sin\phi & \cos\phi & 0 \\ \cos\phi\sin\Theta & \sin\phi\sin\Theta & \cos\Theta \end{bmatrix} \\
M(O^L \to O^S) &= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
M(O^G \to O^L) &= \begin{bmatrix} \cos\phi\cos\Theta & -\sin\phi & \cos\phi\sin\Theta \\ \sin\phi\cos\Theta & \cos\phi & \sin\phi\sin\Theta \\ -\sin\phi & 0 & \cos\Theta \end{bmatrix} \\
M(O^S \to O^L) &= \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
\tag{11}
$$

The problem of a light propagation should be solve using surface system $O^S$, because it is the system common for all layer and is determined only by an incident wave. The system $O^G$ is connected to the given layer by it's optical axis and is constructed in independent way for every layer.

For the wavevector of an incident light $\vec{k_i} = [\beta, 0, k_{iz}]$ in any layer we have two wavevectors: $\vec{k_o} = [\beta, 0, k_{oz}]$ – for ordinary wave, and $\vec{k_e} = [\beta, 0, k_{ez}]$ – for extraordinary one (see Figure 2).

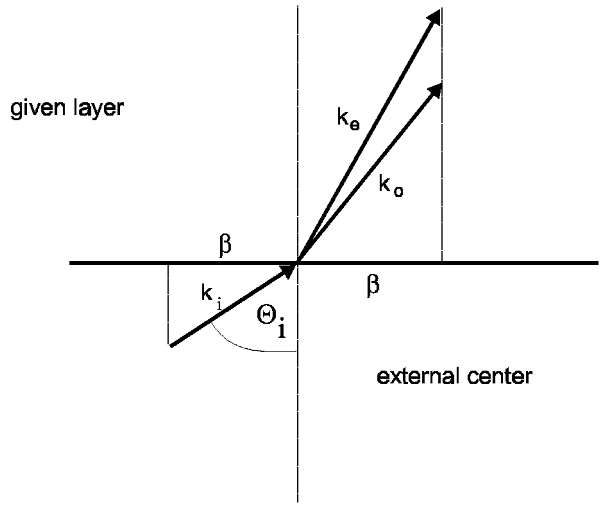


**FIGURE 2** The wavevectors in an external centre and given layer described in system $O^S$.

Certainly, for every layer the relations (in surface system): $k_{ox} = k_{ex} = \beta$ and $k_{ex} = k_{ey} = 0$ have to be fulfilled. $\beta$ can be determined from the initial conditions using an incident angle $\Theta_i$ as:

$$\beta = \frac{\omega}{c} n_i \sin \Theta_i \tag{12}$$

where $n_i$ is refractive index of an external medium.

The wavevectors $\vec{k}_o$ and $\vec{k}_e$ in principle coordinate system of a given layer have the forms:

$$\vec{k}_{oG} = \begin{bmatrix} \beta \cos\alpha \cos\phi \cos\Theta + \beta \sin\alpha \sin\phi \cos\Theta - k_{oz} \sin\Theta \\ -\beta \cos\alpha \sin\phi + \beta \sin\alpha \cos\phi \\ \beta \cos\alpha \cos\phi \sin\Theta + \beta \sin\alpha \sin\phi \sin\Theta + k_{oz} \cos\Theta \end{bmatrix}$$
$$\vec{k}_{eG} = \begin{bmatrix} \beta \cos\alpha \cos\phi \cos\Theta + \beta \sin\alpha \sin\phi \cos\Theta - k_{ez} \sin\Theta \\ -\beta \cos\alpha \sin\phi + \beta \sin\alpha \cos\phi \\ \beta \cos\alpha \cos\phi \sin\Theta + \beta \sin\alpha \sin\phi \sin\Theta + k_{ez} \cos\Theta \end{bmatrix} \tag{13}$$

Using the previous equations one can show that the following relations can be described:

$$k_{oz} = \pm\sqrt{\frac{\omega^2}{c^2} n_o^2 - \beta^2} \qquad k_{ez} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{14}$$

where

$$
\begin{aligned}
A &= n_o^2 \sin^2\Theta + n_e^2\cos^2\Theta \\
B &= \beta \sin 2\Theta \cos(\alpha-\phi)\left(n_e^2 - n_o^2\right) \\
C &= \beta^2\cos^2(\alpha-\phi)\left(n_o^2\cos^2\Theta + n_e^2\sin^2\Theta\right) \\
&\quad + \beta^2\sin^2(\phi-\alpha)n_o^2 - \frac{\omega^2 n_e^2 n_o^2}{c^2}
\end{aligned}
\tag{15}
$$

The relations (14) and (15) give us the information about the wavevectors, which can propagate through given layer for a given incident wave simultaneously in two directions of propagation: into the layer $(+z)$ and back $(-z)$. Therefore one can describe:

$$
\begin{aligned}
\vec{k}_o(+z) &= [\beta, 0, k_{ozT}] \qquad \vec{k}_e(+z) = [\beta, 0, k_{ezT}] \\
\vec{k}_o(-z) &= [\beta, 0, k_{ozR}] \qquad \vec{k}_e(-z) = [\beta, 0, k_{ezR}]
\end{aligned}
\tag{16}
$$

where $k_{ozT}(k_{ezT})$ and $k_{ozR}(k_{ezR})$ denote the $z$-coordinate of wavevector for propagation in $(+z)$ direction–transmitted wave and $(-z)$ direction–reflected wave.

Using the representation of these wavevectors in surface coordinate system and transposition matrixes one can determine the unit vectors of $\vec{E}$ for the both directions of propagation as:

$$
\begin{aligned}
\vec{o}(+z) &= \left[o_{xT}, o_{yT}, o_{zT}\right] \qquad \vec{e}(+z) = \left[e_{xT}, e_{yT}, e_{zT}\right] \\
\vec{o}(-z) &= \left[o_{xR}, o_{yR}, o_{zR}\right] \qquad \vec{e}(-z) = \left[e_{xR}, e_{yR}, e_{zR}\right]
\end{aligned}
\tag{17}
$$

Having the wavevectors and unit vectors of $\vec{E}$ for all layers in the analysed layer system the transmission and reflection coefficients for all boundaries can be calculated. In Figure 3 the schema of wavevectors for given phase boundary drawn in surface coordinate system are presented ($\vec{k}_{oi}$ – incident ordinary wave, $\vec{k}_{ei}$ – incident extraordinary wave, $\vec{k}_{oR}$ – reflected ordinary wave, $\vec{k}_{eR}$ – reflected extraordinary wave, $\vec{k}_{oT}$ – transmitted ordinary wave, $\vec{k}_{eT}$ – transmitted extraordinary wave).

The presented in Figure 3 the wavevectors in a surface coordinate system can be described as:

$$
\begin{aligned}
\vec{k}_{oi} &= [\beta, 0, k_{oiz}]; \quad \vec{k}_{ei} = [\beta, 0, k_{eiz}] \\
\vec{k}_{oT} &= [\beta, 0, k_{oTz}]; \quad \vec{k}_{eT} = [\beta, 0, k_{eTz}] \\
\vec{k}_{oR} &= [\beta, 0, k_{oRz}]; \quad \vec{k}_{eR} = [\beta, 0, k_{eRz}]
\end{aligned}
\tag{18}
$$

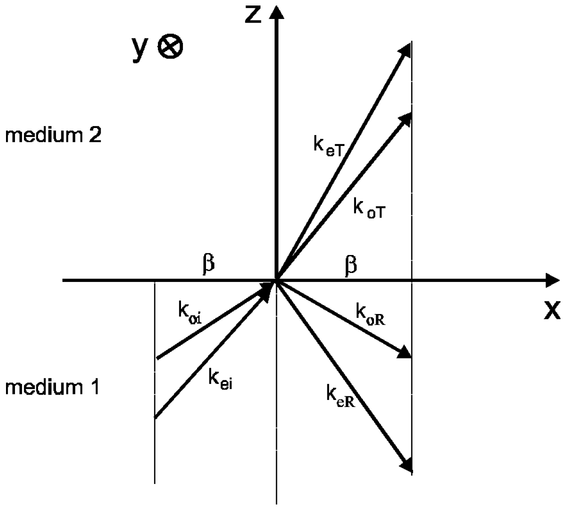


**FIGURE 3** The wavevectors on the phase boundary drawn in surface coordinate system.



and the unit vectors of $\vec{\vec{E}}$ as:

$$\begin{aligned} \vec{o}_i &= [o_{ix}, o_{iy}, o_{iz}]; \quad \vec{e}_i = [e_{ix}, e_{iy}, e_{iz}] \\ \vec{o}_T &= [o_{Tx}, o_{Ty}, o_{Tz}]; \quad \vec{e}_T = [e_{Tx}, e_{Ty}, e_{Tz}] \\ \vec{o}_R &= [o_{Rx}, o_{Ry}, o_{Rz}]; \quad \vec{e}_R = [e_{Rx}, e_{Ry}, e_{Rz}] \end{aligned} \tag{19}$$

Using the other specification (17) we can describe:

- for propagation from 1 to 2 medium

$$\begin{aligned} &\vec{k}_{oi} = \vec{k}_{1o}(+z) \quad \vec{k}_{oT} = \vec{k}_{2o}(+z) \quad \vec{k}_{oR} = \vec{k}_{1o}(-z) \\ &\vec{k}_{ei} = \vec{k}_{1e}(+z) \quad \vec{k}_{eT} = \vec{k}_{2e}(+z) \quad \vec{k}_{eR} = \vec{k}_{1e}(-z) \\ &\vec{o}_i = \vec{o}_1(+z) \quad \vec{o}_T = \vec{o}_2(+z) \quad \vec{o}_R = \vec{o}_1(-z) \\ &\vec{e}_i = \vec{e}_1(+z) \quad \vec{e}_T = \vec{e}_2(+z) \quad \vec{e}_R = \vec{e}_1(-z) \end{aligned} \tag{20}$$

- for propagation from 2 to 1 medium

$$\begin{aligned} &\vec{k}_{oi} = \vec{k}_{2o}(-z) \quad \vec{k}_{oT} = \vec{k}_{1o}(-z) \quad \vec{k}_{oR} = \vec{k}_{2o}(+z) \\ &\vec{k}_{ei} = \vec{k}_{2e}(-z) \quad \vec{k}_{eT} = \vec{k}_{1e}(-z) \quad \vec{k}_{eR} = \vec{k}_{2e}(+z) \\ &\vec{o}_i = \vec{o}_2(-z) \quad \vec{o}_T = \vec{o}_1(-z) \quad \vec{o}_R = \vec{o}_2(+z) \\ &\vec{e}_i = \vec{e}_2(-z) \quad \vec{e}_T = \vec{e}_1(-z) \quad \vec{e}_R = \vec{e}_2(+z) \end{aligned} \tag{21}$$

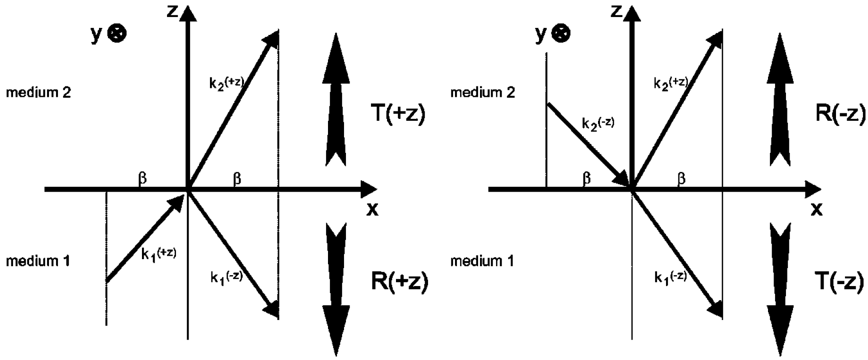


**FIGURE 4** The schema of the results obtained for two data sets.



Using these data one can obtain the information about the wave propagation in $(+z)$ and $(-z)$ direction. The schema of these propagation is presented in Figure 4.

The coefficients T$(+z)$, T$(-z)$, R(+z) and R$(-z)$ denote the transmission and reflection coefficients, respectively, for $(+z)$ and $(-z)$ direction of a wave propagation and can be obtained from Maxwell equations described for incident, transmitted and reflected waves:

$$
\begin{aligned}
\vec{E}_i &= I_o\vec{o}_i e^{i\left(\omega t - \vec{k}_{oi}\vec{r}\right)} + I_e\vec{e}_i e^{i\left(\omega t - \vec{k}_{ei}\vec{r}\right)} \\
\vec{E}_R &= R_o\vec{o}_R e^{i\left(\omega t - \vec{k}_{oR}\vec{r}\right)} + R_e\vec{e}_R e^{i\left(\omega t - \vec{k}_{eR}\vec{r}\right)} \\
\vec{E}_T &= T_o\vec{o}_T e^{i\left(\omega t - \vec{k}_{oT}\vec{r}\right)} + T_e\vec{e}_T e^{i\left(\omega t - \vec{k}_{eT}\vec{r}\right)}
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\vec{H}_i &= \frac{1}{\omega\mu_0\mu}\left[\vec{k}_{oi} \times \left(I_o\vec{o}_i e^{i\left(\omega t - \vec{k}_{oi}\vec{r}\right)}\right) + \vec{k}_{ei} \times \left(I_e\vec{e}_i e^{i\left(\omega t - \vec{k}_{ei}\vec{r}\right)}\right)\right] \\
\vec{H}_R &= \frac{1}{\omega\mu_0\mu}\left[\vec{k}_{oR} \times \left(R_o\vec{o}_R e^{i\left(\omega t - \vec{k}_{oR}\vec{r}\right)}\right) + \vec{k}_{eR} \times \left(R_e\vec{e}_R e^{i\left(\omega t - \vec{k}_{eR}\vec{r}\right)}\right)\right] \\
\vec{H}_T &= \frac{1}{\omega\mu_0\mu}\left[\vec{k}_{oT} \times \left(T_o\vec{o}_T e^{i\left(\omega t - \vec{k}_{oT}\vec{r}\right)}\right) + \vec{k}_{eT} \times \left(T_e\vec{e}_T e^{i\left(\omega t - \vec{k}_{eT}\vec{r}\right)}\right)\right]
\end{aligned}
\tag{23}
$$

where $I_o, I_e, T_o, T_e, R_o$ and $R_e$ denote the amplitudes for incident, transmitted and reflected waves (for ordinary and extraordinary ones, respectively).

For boundary the Eqs. (22) and (23) assume the forms:

$$\begin{cases} 1)\ I_o o_{ix} + I_e e_{ix} + R_o o_{Rx} + R_e e_{Rx} = T_o o_{Tx} + T_e e_{Rx} \\ 2)\ I_o o_{iy} + I_e e_{iy} + R_o o_{Ry} + R_e e_{Ry} = T_o o_{Ty} + T_e e_{Ry} \\ 3)\ I_o k_{oiz} o_{iy} + I_e k_{eiz} e_{iy} + R_o k_{oRz} o_{Ry} + R_e k_{eRz} e_{Ry} \\ \quad = T_o k_{oTz} o_{Ty} + T_e k_{eTz} e_{Ty} \\ 4)\ I_o(k_{oiz} o_{ix} - \beta o_{iz}) + I_e(k_{eiz} e_{ix} - \beta e_{iz}) + R_o(k_{oRz} o_{Rx} - \beta o_{Rz}) \\ \quad + R_e(k_{eRz} e_{Rx} - \beta e_{Rz}) \\ \qquad = T_o(k_{oTz} o_{Tx} - \beta o_{Tz}) + T_e(k_{eTz} e_{Tx} - \beta e_{Tz}) \end{cases} \tag{24}$$

After an applying the following substitutions:

$$\begin{array}{ll} A_o^T = o_{ix} e_{Ry} - o_{iy} e_{Rx};\ C_o^T = o_{Tx} e_{Ry} - o_{Ty} e_{Rx} & S_1 = k_{oiz} o_{iy} \\ B_o^T = e_{ix} e_{Ry} - e_{iy} e_{Rx};\ D_o^T = e_{Tx} e_{Ry} - e_{Ty} e_{Rx} & S_2 = k_{eiz} e_{iy} \\ M_o^T = o_{Rx} e_{Ry} - o_{Ry} e_{Rx} & S_3 = k_{oRz} o_{Ry} \\ & S_4 = k_{eRz} e_{Ry} \\ & S_5 = k_{oTz} o_{Ty} \\ & S_6 = k_{eTz} e_{Ty} \end{array}$$

$$\begin{array}{ll} A_e^T = o_{ix} o_{Ry} - o_{iy} o_{Rx};\ C_e^T = o_{Tx} o_{Ry} - o_{Ty} o_{Rx} & Z_1 = k_{oiz} o_{ix} - \beta o_{iz} \\ B_e^T = e_{ix} o_{Ry} - e_{iy} o_{Rx};\ D_e^T = e_{Tx} o_{Ry} - e_{Ty} o_{Rx} & Z_2 = k_{eiz} e_{ix} - \beta e_{iz} \\ M_e^T = e_{Rx} o_{Ry} - e_{Ry} o_{Rx} & Z_3 = k_{oRz} o_{Rx} - \beta o_{Rz} \\ & Z_4 = k_{eRz} e_{Rx} - \beta e_{Rz} \\ & Z_5 = k_{oTz} o_{Tx} - \beta o_{Tz} \\ & Z_6 = k_{eTz} e_{Tx} - \beta e_{Tz} \end{array} \tag{25}$$

and the next substitutions:

$$\begin{aligned} X_o^{ST} &= S_1 M_o^T M_e^T - S_3 A_o^T M_e^T - S_4 A_e^T M_o^T \\ X_e^{ST} &= S_2 M_o^T M_e^T - S_3 B_o^T M_e^T - S_4 B_e^T M_o^T \\ Y_o^{ST} &= S_5 M_o^T M_e^T - S_3 C_o^T M_e^T - S_4 C_e^T M_o^T \\ Y_e^{ST} &= S_6 M_o^T M_e^T - S_3 D_o^T M_e^T - S_4 D_e^T M_o^T \\ X_o^{ZT} &= Z_1 M_o^T M_e^T - Z_3 A_o^T M_e^T - Z_4 A_e^T M_o^T \\ X_e^{ZT} &= Z_2 M_o^T M_e^T - Z_3 B_o^T M_e^T - Z_4 B_e^T M_o^T \\ Y_o^{ZT} &= Z_5 M_o^T M_e^T - Z_3 C_o^T M_e^T - Z_4 C_e^T M_o^T \\ Y_e^{ZT} &= Z_6 M_o^T M_e^T - Z_3 D_o^T M_e^T - Z_4 D_e^T M_o^T \end{aligned} \tag{26}$$

one can obtain:

$$
\begin{aligned}
T_o &= \frac{X_o^{ST}Y_e^{ZT} - Y_e^{ST}X_o^{ZT}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} I_o + \frac{X_e^{ST}Y_e^{ZT} - Y_e^{ST}X_e^{ZT}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} I_e \\
T_e &= \frac{X_o^{ZT}Y_o^{ST} - Y_o^{ZT}X_o^{ST}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} I_o + \frac{X_e^{ZT}Y_o^{ST} - Y_o^{ZT}X_e^{ST}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} I_e
\end{aligned}
\tag{27}
$$

what can be described in the other form as:

$$T_o = t_{oo}I_o + t_{eo}I_e \qquad T_e = t_{oe}I_o + t_{ee}I_e \tag{28}$$

where

$$
\begin{aligned}
t_{oo} &= \frac{X_o^{ST}Y_e^{ZT} - Y_e^{ST}X_o^{ZT}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} \qquad t_{eo} = \frac{X_e^{ST}Y_e^{ZT} - Y_e^{ST}X_e^{ZT}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} \\
t_{oe} &= \frac{X_o^{ZT}Y_o^{ST} - Y_o^{ZT}X_o^{ST}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}} \qquad t_{ee} = \frac{X_e^{ZT}Y_o^{ST} - Y_o^{ZT}X_e^{ST}}{Y_e^{ZT}Y_o^{ST} - Y_e^{ST}Y_o^{ZT}}
\end{aligned}
\tag{29}
$$

Assuming the light wave in the Jones complex vector $I = \begin{bmatrix} I_o \\ I_e \end{bmatrix} e^{i\omega t}$, where $I_o$ and $I_e$ denote the complex amplitudes of electric field for ordinary and extraordinary waves, respectively, the transmission coefficient can be written in matrix form as:

$$T = \begin{bmatrix} t_{oo} & t_{eo} \\ t_{oe} & t_{ee} \end{bmatrix} \tag{30}$$

In the similar method one can obtain the reflection coefficient:

$$R = \begin{bmatrix} r_{oo} & r_{eo} \\ r_{oe} & r_{ee} \end{bmatrix} \tag{31}$$

where

$$
\begin{aligned}
r_{oo} &= \frac{X_o^{SR}Y_e^{ZR} - Y_e^{SR}X_o^{ZR}}{Y_e^{ZR}Y_o^{SR} - Y_e^{SR}Y_o^{ZR}} \qquad r_{eo} = \frac{X_e^{SR}Y_e^{ZR} - Y_e^{SR}X_e^{ZR}}{Y_e^{ZR}Y_o^{SR} - Y_e^{SR}Y_o^{ZR}} \\
r_{oe} &= \frac{X_o^{ZR}Y_o^{SR} - Y_o^{ZR}X_o^{SR}}{Y_e^{ZR}Y_o^{SR} - Y_e^{SR}Y_o^{ZR}} \qquad r_{ee} = \frac{X_e^{ZR}Y_o^{SR} - Y_o^{ZR}X_e^{SR}}{Y_e^{ZR}Y_o^{SR} - Y_e^{SR}Y_o^{ZR}}
\end{aligned}
\tag{32}
$$

and

$$
\begin{aligned}
X_o^{SR} &= S_1M_o^RM_e^R - S_5A_o^RM_e^R - S_6A_e^RM_o^R \\
X_e^{SR} &= S_2M_o^RM_e^R - S_5B_o^RM_e^R - S_6B_e^RM_o^R \\
Y_o^{SR} &= -S_3M_o^RM_e^R + S_5C_o^RM_e^R + S_6C_e^RM_o^R \\
Y_e^{SR} &= -S_4M_o^RM_e^R + S_5D_o^RM_e^R + S_6D_e^RM_o^R
\end{aligned}
$$

$$
\begin{aligned}
A_o^R &= o_{ix}e_{Ty} - o_{iy}e_{Tx}; \quad C_o^R = o_{Rx}e_{Ty} - o_{Ry}e_{Tx} \\
B_o^R &= e_{ix}e_{Ty} - e_{iy}e_{Tx}; \quad D_o^R = e_{Rx}e_{Ty} - e_{Ry}e_{Tx} \\
M_o^R &= o_{Tx}e_{Ty} - o_{Ty}e_{Tx}
\end{aligned}
$$

$$
\begin{aligned}
X_o^{ZR} &= Z_1 M_o^R M_e^R - Z_5 A_o^R M_e^R - Z_6 A_e^R M_o^R \\
X_e^{ZR} &= Z_2 M_o^R M_e^R - Z_5 B_o^R M_e^R - Z_6 B_e^R M_o^R \\
Y_o^{ZR} &= -Z_3 M_o^R M_e^R + Z_5 C_o^R M_e^R + Z_6 C_e^R M_o^R \\
Y_e^{ZR} &= -Z_4 M_o^R M_e^R + Z_5 D_o^R M_e^R + Z_6 D_e^R M_o^R
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
A_e^R &= o_{ix} o_{Ty} - o_{iy} o_{Tx}; \quad C_e^R = o_{Rx} o_{Ty} - o_{Ry} o_{Tx} \\
B_e^R &= e_{ix} o_{Ty} - e_{iy} o_{Tx}; \quad D_e^R = e_{Rx} o_{Ty} - e_{Ry} o_{Tx} \\
M_e^R &= e_{Tx} o_{Ty} - e_{Ty} o_{Tx}
\end{aligned}
$$

After these calculations the transmission and reflection coefficients for the both direction of a wave propagation are obtained. In the further analysis the following descriptions are applied:

$T_{(n,n+1)}$ – transmission for direction of a wave propagation $(+z)$;
$T'_{(n,n+1)}$ – transmission for direction of a wave propagation $(-z)$;
$R_{(n,n+1)}$ – reflection for direction of a wave propagation $(+z)$;
$R'_{(n,n+1)}$ – reflection for direction of a wave propagation $(-z)$.

The similar convention are applied to the wavevectors and unit vectors of electric filed e.g., $\vec{k}'_o = \vec{k}_o(-z)$ etc. (symbol *prim* will be applied to the $(-z)$ direction).

The transmission and reflection coefficients obtained above describe the wavevectors lied in the way which is shown in Figure 3, but after passing through the first boundary (point A), the waves generated in layer no. $(n)$ do not achieve the same point on the second boundary (see Fig. 5), because an ordinary wave achieves point B, but extraordinary one achieve the other point. The point B is achieved by an extraordinary wave generated in point C.

Let a wave in point A in a layer $(n-1)$ has a form $\vec{E}_A^{(n-1)} = \begin{bmatrix} I_o \\ I_e \end{bmatrix} e^{i\omega t}$. In a layer no. $(n)$ this wave has a form:

$$
\vec{E}_A^{(n)} = T_{(n-1,n)} \begin{bmatrix} I_o \\ I_e \end{bmatrix} e^{i\omega t} = \begin{bmatrix} t_{oo}^{(n-1,n)} I_o + t_{eo}^{(n-1,n)} I_e \\ t_{oe}^{(n-1,n)} I_o + t_{ee}^{(n-1,n)} I_e \end{bmatrix} e^{i\omega t} \tag{34}
$$

The ordinary wave in layer $(n)$ can be written as:

$$
\vec{E}_o^{(n)} = \left( t_{oo}^{(n-1,n)} I_o + t_{eo}^{(n-1,n)} I_e \right) \vec{o}^{(n)} \exp\left[ i \left( \omega t - \vec{k}_o^{(n)} \circ \vec{r} \right) \right] \tag{35}
$$

where in surface coordinate system $\vec{k}_o^{(n)} = \left[ \beta, 0, k_{oz}^{(n)} \right]$. The vector $\vec{r}$ describes the way from point A to point B and it have in the same

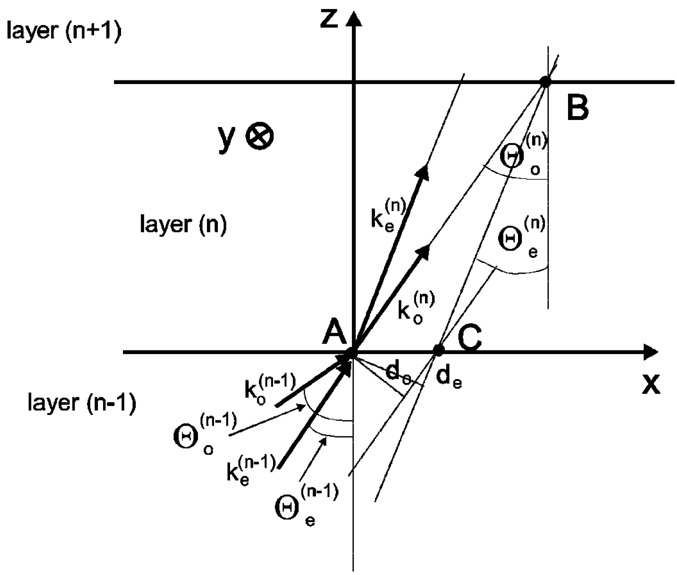


**FIGURE 5** The schema of a wave propagation through an anisotropic layer.

coordinate system a form: $\vec{r} = \left[h^{(n)} tg\Theta_o^{(n)}, 0, h^{(n)}\right]$, where $h^{(n)}$–thickness of a layer $(n)$. Therefore we have: $\vec{k}_o^{(n)} \circ \vec{r} = \beta h^{(n)} tg\Theta_o^{(n)} + k_{oz}^{(n)} h^{(n)}$.

Taking advantage of a relation $\beta/k_{oz}^{(n)} = tg\Theta_o^{(n)}$ one can obtain $\vec{k}_o^{(n)} \circ \vec{r} = \left(\beta^2 h^{(n)}/k_{oz}^{(n)}\right) + k_{oz}^{(n)} h^{(n)}$. As one can see the phase shift for ordinary wave is equal $\delta_o^{(n)} = \left(\beta^2 h^{(n)}/k_{oz}^{(n)}\right) + k_{oz}^{(n)} h^{(n)}$.

In point B we can describe an ordinary wave in a form:

$$\vec{E}_{oB}^{(n)} = \left(t_{oo}^{(n-1,n)} I_o + t_{eo}^{(n-1,n)} I_e\right) \vec{o}^{(n)} \exp\left[i\left(\omega t - \delta_o^{(n)}\right)\right] \tag{36}$$

The extraordinary wave, which achieves a point B is generated by ordinary and extraordinary waves achieved in layer ($n$–1) point C. One can notice that the following relation is fulfilled:

$$AC = h^{(n)}\left(tg\Theta_o^{(n)} - tg\Theta_e^{(n)}\right) \tag{37}$$

where $tg\Theta_o^{(n)} = \beta/k_{oz}^{(n)}$ and $tg\Theta^{(n)} = \beta/k_{ez}^{(n)}$.

It can be shown, that if a wave in layer ($n$–1) achieving point A is given by (34), then point C is achieved by a wave:

$$\vec{E}_C^{(n-1)} = \begin{bmatrix} I_o \\ I_e \end{bmatrix} e^{-i\delta_p} \tag{38}$$

where $\delta_p = \beta^2 h^{(n)}\left((1/k_{oz}^{(n)}) - (1/k_{ez}^{(n)})\right)$.

In the $(n)$ layer the following wave is generated:

$$\overrightarrow{E}_C^{(n)} = T_{(n-1,n)} \begin{bmatrix} I_o \\ I_e \end{bmatrix} e^{-i\delta_p} = \begin{bmatrix} t_{oo}^{(n-1,n)} I_o + t_{eo}^{(n-1,n)} I_e \\ t_{oe}^{(n-1,n)} I_o + t_{ee}^{(n-1,n)} I_e \end{bmatrix} e^{-i\delta_p} \tag{39}$$

After the passing of an extraordinary wave from point C to point B, we have an extraordinary wave in a layer $(n)$ (in point B) as:

$$\overrightarrow{E}_{eB}^{(n)} = \left( t_{oe}^{(n-1,n)} I_o + t_{ee}^{(n-1,n)} I_e \right) \vec{e}^{(n)} \exp\left[ i \left( \omega t - \delta_e^{(n)} \right) \right] \tag{40}$$

where $\delta_e^{(n)} = \left( \beta^2 h^{(n)} / k_{oz}^{(n)} \right) + h^{(n)} k_{ez}^{(n)}$.

The final wave achieving point B in a layer $(n)$ can be written as:

$$\overrightarrow{E}_B^{(n)} = \Delta_{(n)} T_{(n-1,n)} \overrightarrow{E}_A^{(n-1)} \tag{41}$$

where

$$\Delta_{(n)} = \begin{bmatrix} \exp(-i\delta_o^{(n)}) & 0 \\ 0 & \exp(-i\delta_e^{(n)}) \end{bmatrix} \tag{42}$$

Analyzing the next reflections one can show, that the phase shifts obtained for the consecutive beams is equal:

$$\delta_d = \beta^2 h^{(n)} \left( \frac{1}{k_{oz}^{(n)}} - \frac{1}{k_{oz}'^{(n)}} \right) \tag{43}$$

The same analysis can be done for direction $(-z)$ and for reflection. The results and conclusions are very similar.

As one can see the phase shifts caused by $x$-coordinates of wavevectors are identically. For this reason it can be omitted and for $(+z)$ and $(-z)$ directions one can write:

$$\begin{array}{ll} \delta_o^{(n)} = h^{(n)} k_{oz}^{(n)} & \delta_o'^{(n)} = -h^{(n)} k_{oz}'^{(n)} \\ \delta_e^{(n)} = h^{(n)} k_{ez}^{(n)} & \delta_e'^{(n)} = -h^{(n)} k_{ez}'^{(n)} \end{array} \tag{44}$$

Having the transmission and reflection coefficients for all boundaries into the layer system for the both direction of a wave propagation the optical coefficients for the layers can be determined.

For a layer $(n)$ these coefficients are denoted by $T_{(n)}$, $T'_{(n)}$, $R_{(n)}$ and $R'_{(n)}$, and mean:

- $T_{(n)}$ – transmission coefficient for a layer $(n)$ and direction of a wave propagation $(+z)$;
- $T'_{(n)}$ – transmission coefficient for a layer $(n)$ and direction of a wave propagation $(-z)$;

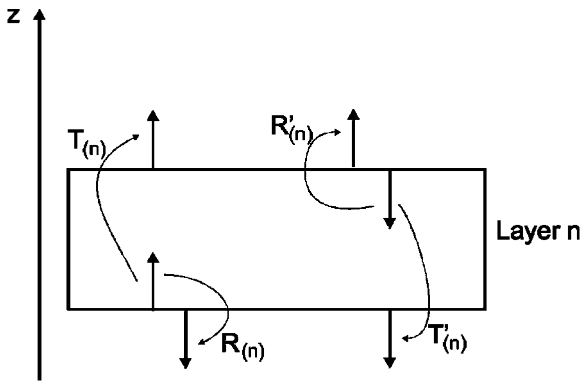


**FIGURE 6** The schema of determining of transmission and reflection coefficients for single layer.



- $R_{(n)}$ – reflection coefficient for a layer $(n)$ and direction of a wave propagation $(+z)$;
- $R'_{(n)}$ – reflection coefficient for a layer $(n)$ and direction of a wave propagation $(-z)$.

These coefficients describe the wave propagation from interior to exterior of a layer (Fig. 6).

- transmission coefficient for direction of a wave propagation $(+z)$ If a wave in layer $(n)$, next to boundary with a layer $(n+1)$ is given as $\vec{E}$, then a wave entering into medium $(n+1)$ can be written as:

$$\vec{E}_T = \sum_{m=1}^{\infty} \vec{E}_M \tag{45}$$

where $m$ denotes the number of waves taking into account in the interference process and:

$$\vec{E}_m = T_{(n,n+1)} \Delta_{1(n)}^{m-1} \Delta_{(n)} \vec{E} \tag{46}$$

where $\Delta_{1(n)} = \Delta_{(n)} R'_{(n-1,n)} \Delta'_{(n)} R_{(n,n+1)}$.

Finally, this transmission coefficient for a layer $(n)$ is given by the matrix:

$$T_{(n)} = \sum_{m=1}^{\infty} \left[ T_{(n,n+1)} \Delta_{1(n)}^{m-1} \Delta_{(n)} \right] = \begin{bmatrix} T_{oo}^{(n)} & T_{eo}^{(n)} \\ T_{oe}^{(n)} & T_{ee}^{(n)} \end{bmatrix} \tag{47}$$

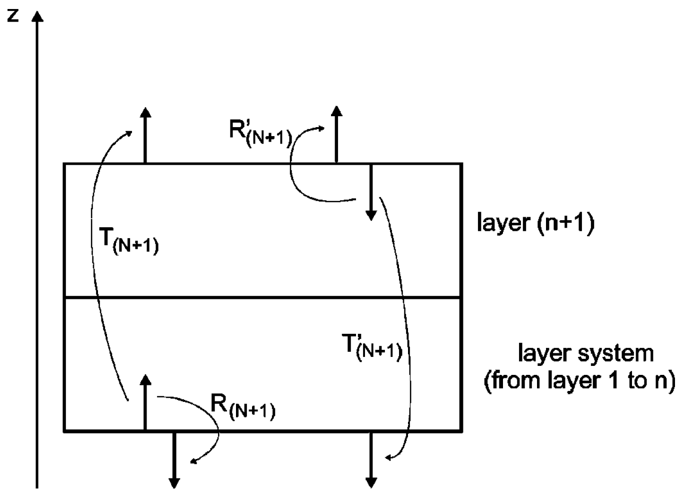


**FIGURE 7** The schema presenting the way to determine the optical parameters of a layer system.



The other coefficients one can found, that:

- transmission for direction of a wave propagation $(-z)$

$$T'_{(n)} = \sum_{m=1}^{\infty}\left[T'_{(n-1,n)}\Delta_{2(n)}^{m-1}\Delta'_{(n)}\right] = \begin{bmatrix} T'^{(n)}_{oo} & T'^{(n)}_{eo} \\ T'^{(n)}_{oe} & T'^{(n)}_{ee} \end{bmatrix} \tag{48}$$

where $\Delta_{2(n)} = \Delta'_{(n)}R_{(n,n+1)}\Delta_{(n)}R'_{(n-1,n)}$.

- reflection for direction of a wave propagation $(+z)$

$$R_{(n)} = \sum_{m=2}^{\infty}\left[T'_{(n-1,n)}\Delta_{2(n)}^{m-2}\Delta'_{(n)}R_{(n,n+1)}\Delta_{(n)}\right] = \begin{bmatrix} R^{(n)}_{oo} & R^{(n)}_{eo} \\ R^{(n)}_{oe} & R^{(n)}_{ee} \end{bmatrix} \tag{49}$$

- reflection for direction of a wave propagation $(-z)$

$$R'_{(n)} = \sum_{m=2}^{\infty}\left[T_{(n,n+1)}\Delta_{1(n)}^{m-2}\Delta_{(n)}R'_{(n-1,n)}\Delta'_{(n)}\right] = \begin{bmatrix} R'^{(n)}_{oo} & R'^{(n)}_{eo} \\ R'^{(n)}_{oe} & R'^{(n)}_{ee} \end{bmatrix} \tag{50}$$

The coefficients presented above determined for all layers make it possible to obtain the similar coefficients for all layer system.

Let's improve $T_{(N)}, T'_{(N)}, R_{(N)}$ and $R'_{(N)}$ coefficients which denote the transmission and reflection indices for a layer system (from layer (1) to layer $(n)$) describing the way of a light from interior to exterior of a system. The coefficients $T_{(N+1)}, T'_{(N+1)}, R_{(N+1)}$ and $R'_{(N+1)}$ (Fig. 7), which characterize the layer system from layer (1) to layer $(n+1)$ can be described as:

- transmission for direction of a wave propagation $(+z)$

$$T_{(N+1)} = \sum_{m=1}^{\infty} \left[ T_{(n+1)} \left( R'_{(N)} R_{(n+1)} \right)^{m-1} T_{(N)} \right] = \begin{bmatrix} T_{oo}^{(N+1)} & T_{eo}^{(N+1)} \\ T_{oe}^{(N+1)} & T_{ee}^{(N+1)} \end{bmatrix} \quad (51)$$

- transmission for direction of a wave propagation $(-z)$

$$T'_{(N+1)} = \sum_{m=1}^{\infty} \left[ T'_{(N)} \left( R_{(n+1)} R'_{(N)} \right)^{m-1} T'_{(n+1)} \right] = \begin{bmatrix} T'^{(N+1)}_{oo} & T'^{(N+1)}_{eo} \\ T'^{(N+1)}_{oe} & T'^{(N+1)}_{ee} \end{bmatrix} \quad (52)$$

reflection for direction of a wave propagation $(+z)$

$$\begin{aligned} R_{(N+1)} &= R_{(N)} + \sum_{m=2}^{\infty} \left[ T'_{(N)} \left( R_{(n+1)} R'_{(N)} \right)^{m-2} R_{(n+1)} T_{(N)} \right] \\ &= \begin{bmatrix} R_{oo}^{(N+1)} & R_{eo}^{(N+1)} \\ R_{oe}^{(N+1)} & R_{ee}^{(N+1)} \end{bmatrix} \end{aligned} \quad (53)$$

- reflection for direction of a wave propagation $(-z)$

$$\begin{aligned} R'_{(N+1)} &= R'_{(n+1)} + \sum_{m=2}^{\infty} \left[ T_{(n+1)} \left( R'_{N} R_{(n+1)} \right)^{m-2} R'_{(N)} T'_{(n+1)} \right] \\ &= \begin{bmatrix} R'^{(N+1)}_{oo} & R'^{(N+1)}_{eo} \\ R'^{(N+1)}_{oe} & R'^{(N+1)}_{ee} \end{bmatrix} \end{aligned} \quad (54)$$

In this way the all needed coefficients describing the optical properties of all layer system can be obtained.

## 3. FINAL CONCLUSIONS

The theory presented above makes us possible to work out the computer program called CSOP (Computer Support of Optimization Process). This program can be use to do very fast optimization processes for color liquid crystal display. Additionally, it can be used as a support to determine the optical parameters of any layer needed to construct the display with given optical parameters. The verification procedure comparing the experimental and calculated results for standard TN display were carried out, and the obtained results showed that the difference between these results are not higher than $2 - 3\%$. It confirm, that this model is very good approximation of real circumstances.

This program can be used to study of a color displays which characterization is particularly difficult using other method.

## REFERENCES

[1] Ratajczyk, F. (1994). *Optyka Ośrodków Anizotropowych*, PWN: Warsaw.
[2] Yeh, P. & Gu, C. (1999). *Optics of Liquid Crystal Displays*, John Wiley and Sons: New York.
[3] Born, M. & Wolf, E. (2003). *Principle of Optics*, Cambridge University Press.
[4] Bahadur, B. (1992). *Liquid Crystals Applications and Uses*, World Scientific.